DETERMINING THE AVERAGE FLOW CHARACTERISTICS OF THE CARRIER FLUID IN A TURBULENT GASEOUS SUSPENSION

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The effect of solid particles on the thickness of the viscous sublayer in a stream of a gaseous suspension is analyzed on the basis of the two-layer flow model. The characteristics of the velocity profile of the carrier gas are determined, with collision interaction between solid particles and channel walls taken into account.

Studying the effect of solid particles on the average flow characteristics of a carrier fluid is very necessary not only for the solution of problems in pneumatic transport (for example, determination of the critical velocity) but also for calculating the rates of various transport processes which occur in streams of gaseous suspensions. Thus, knowing the effect of solid particles on the viscous sublayer thickness δ is of fundamental importance in an analysis of the mechanism by which the rate of heat transfer in the gaseous suspension increases. Although it has been postulated by some [1] that the increase in the Nu_s/Nu ratio reflects primarily a decrease in δ during an injection of solid particles, in most studies [2-5] this effect is disregarded. The reason for this is, above all, an almost complete lack of test data on the thickness of boundary layers in gaseous suspension streams.

Let us assume that, as a solid particle enters the stream, the radial profile of velocity in a continuum (the linear and the logarithmic profiles respectively in the viscous sublayer and in the mainstream) is retained. Using the two-layer model as a basis and considering that the coefficients in these equations generally are not identical to the corresponding coefficients in the equations for a stream of pure fluid, we will represent the flow characteristics as follows:

1) for the turbulent mainstream with a molal mass transfer mechanism

$$0 \leqslant r < r_1; \quad \nu^* \gg \nu; \qquad v = v^* \left(\frac{v^* \delta}{\nu} + \frac{1}{\chi} \ln \frac{r_0 - r}{\delta} \right); \tag{1}$$

2) for the viscous sublayer with a molecular mechanism

$$r_1 \leqslant r \leqslant r_0; \quad \nu^* \ll \nu; \qquad v = \frac{v^{*2} (r_0 - r)}{\nu}. \tag{2}$$

For a calculation of the velocity profile according to (1) and (2), we need data on the effective value of the Karman constant χ, on the dynamic characteristics of the carrier medium (v^*) in the gaseous suspension, and on the thickness of the viscous sublayer. For an estimate of χ we will use the relations suggested in [6]:

$$\chi = 0.36 (1 + \mu)^{0.25}; \quad \mu < 12; \qquad \chi = 0.11 (1 + \mu)^{0.8}; \quad \mu > 12. \tag{3}$$

It is difficult, however, to interpret the physical causes of an abrupt change in the χ/χ_0 characteristic at $\mu = 12$. Apparently, the data presentation in terms of formulas (3) implies that the mass rate of gas

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and the mass rate of solid particles contribute equally to the eddy viscosity and the resulting turbulent shearing stress. It would be interesting to evaluate the data in the form $\chi/\chi_0 = 1 + m\mu^n$ with the possibility of accounting for unequal contributions by the two components. Such an evaluation will yield

$$\chi/\chi_0 = 1 + 0.16\,\mu^{0.9}. \tag{4}$$

The discontinuity at $\mu = 12$ and thus the essential incongruity between the two relations in (3) will have disappeared.

For estimating the transient velocity of the carrier medium, it is necessary to determine the net quantity ξ characterizing the frictional stress in the fluid due to suspended particles without the explicit contribution made by direct collision interaction with the wall: $\xi = \xi_S - \xi_C$, where $\xi_C = 2\Delta p_C D/\rho v^2 L$. Assuming a collision of every particle with the wall equally probable, such an assumption being especially valid in the case of a vertical stream, we can obtain for the collision frequency

$$n = \frac{1}{\pi DL} \cdot \frac{\pi D^2 L}{4V_p}\,\beta\,\frac{\pi v'_{p+}}{2D} = \frac{\pi}{8} \cdot \frac{\beta v'_{p+}}{V_p}. \tag{5}$$

The pressure loss per collision is then

$$\Delta p_C = n\pi DLm_p v_{pw}\zeta\frac{4}{\pi D^2} = \frac{\pi\zeta}{2} \cdot \frac{L}{D}\,\rho_p v_{pw} v'_{p+}\beta, \tag{6}$$

where ζ denotes an empirical loss factor of longitudinal velocity and v_{pw} denotes the velocity of solid particles at the instant of collision.

Unlike in [7], the mechanism of collision interaction with the wall according to (6) is not converted into quasifriction. Moreover, the quantity v'_{p+} should be determined according to the procedure in [8], the latter being more rigorous than that in [7]. Since for a pure fluid one may assume that $v'_{0+} \propto v_0^* = v\sqrt{\xi_0/8}$, hence

$$\xi_C = \pi\zeta\varphi'_+\frac{v'_+}{v'_{0+}}\sqrt{\frac{\xi_0}{8}} \cdot \frac{v_{pw}}{v_p}\,\mu. \tag{7}$$

Here φ'_+ takes into account the sliding of the components at the fluctuation rate and is determined according to the formulas in [8], while v'_+/v'_{0+} accounts for the effect of solid particles on the turbulence of the carrier medium as described in [9].

An essential feature of the expression derived here is that the longitudinal velocity of solid particles at the instant of collision appears in it. We will evaluate this quantity (v_{pw}) on the basis of the velocity of the fluid at a distance from the wall equal to the diameter of one particle and on the basis of the relative velocity of the solid particles. For coarse particles ($d_p > \delta$) one may let $v_{pw} = v_p$, moreover, on account of the steepness of the logarithmic profile and the high inertia of these particles. For fine particles ($d_p < \delta$), with a low inertia,

$$v_{pw} \simeq \frac{v^{*2}\,d_p}{\nu} = v\,\mathrm{Re}\,\frac{d_p}{D} \cdot \frac{\xi}{8}, \tag{8}$$

because the velocity of ascent is known to be lower than that. Then, we have for coarse particles

$$\frac{\xi}{\xi_0} = \frac{\xi_S - \xi_C}{\xi_0} = \frac{\xi_S}{\xi_0} - \frac{\pi\zeta\varphi'_+}{\sqrt{8\xi_0}} \cdot \frac{v'_+}{v'_{0+}}\,\mu \tag{9}$$

and for fine particles

$$\frac{\xi}{\xi_0} = \frac{\xi_S}{\xi_0} - \frac{\pi\zeta\varphi'_+}{\sqrt{8\xi_0}} \cdot \frac{v'_+}{v'_{0+}}\,\mathrm{Re}\,\frac{d_p}{D} \cdot \frac{\xi}{8}\,\mu, \tag{10}$$

wherefrom

$$\frac{\xi}{\xi_0} = \frac{\xi_S}{\xi_0}\left[1 + \frac{\pi\zeta\varphi'_+\mathrm{Re}}{8}\,\frac{v'_+}{v'_{0+}} \cdot \frac{d_p}{D}\sqrt{\frac{\xi_0}{8}}\,\mu\right]^{-1}. \tag{11}$$

Formulas (9) and (11) make it possible to convert total pressure losses to parameters ξ and v^*.

We will now determine, without any additional assumptions, the thickness of the viscous sublayer in a suspension stream. We take (1) and (2) into consideration as well as the following integral identities:

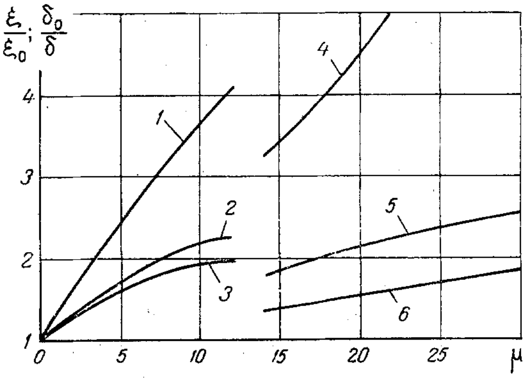

Fig. 1. Coefficients of hydraulic drag and of viscous sublayer thickness under test conditions in [10] (1, 2, 3) and under test conditions in [11] (4, 5, 6): ξ_S/ξ_0 (1, 4), ξ/ξ_0 (2, 5), δ_0/δ (3, 6).

$$\frac{\bar{v}}{v^*} = \sqrt{\frac{8}{\xi}} = \frac{2}{r_0^2}\int_0^{r_0} \frac{v}{v^*} r dr = \frac{2}{r_0^2}\int_{r_0-\delta}^{r_0} \frac{v^*}{\nu}(r_0 - r) r dr$$
$$+ \frac{2}{r_0^2}\int_0^{r_0-\delta} \left(\frac{v^*\delta}{\nu} + \frac{1}{\chi}\ln\frac{r_0 - r}{\delta}\right) r dr. \tag{12}$$

Integrating (12) and collecting like terms in powers of δ/r_0 will yield

$$\sqrt{\frac{8}{\xi}} = \frac{1}{\chi}\ln\frac{r_0}{\delta} - \frac{1}{\chi}\cdot\frac{3}{2} + \frac{\delta}{r_0}\left(\frac{v^* r_0}{\nu} + \frac{2}{\chi}\right)$$
$$- \left(\frac{\delta}{r_0}\right)^2\left(\frac{v^* r_0}{\nu} + \frac{1}{2\chi}\right) + \left(\frac{\delta}{r_0}\right)^3\left(\frac{v^* r_0}{3\nu}\right). \tag{13}$$

The first three terms on the right-hand side of equality (13) are much larger than the remaining ones. Thus, for a pure stream with a Reynolds number Re = 10,000 the first three terms constitute 97% of the total value. With a higher Reynolds number and a solid phase dispersed in the gas stream, the decrease in δ justifies it even more to disregard the remaining terms. In this way,

$$\ln\frac{r_0}{\delta} \simeq \left(\chi\sqrt{\frac{8}{\xi}} + \frac{3}{2}\right) - \chi\,\mathrm{Re}\sqrt{\xi/32}\,\frac{\delta}{r_0}. \tag{14}$$

The solution to this transcendental equation, which can be obtained rather easily by the graphical method, yields a formula for δ.

As an example, we have calculated the hydrodynamic characteristics of a carrier medium flowing under the test conditions in [10, 11]: a gaseous suspension of graphite with d_p = 1-5 μ and Re = 25,000. The test data in [10] cover mass flow concentrations $\mu < 12$ and the test data in [11] cover mass flow concentrations μ = 10-30. Some discrepancy between both test data within the $10 < \mu < 15$ range can, apparently, be explained by a definite variance in the diameters of the particles and by the difference in test procedures. An analysis of calculated results shown in Fig. 1 indicates an appreciable difference between ξ and ξ_S as well as ξ_0. This means that the approximations $\xi = \xi_0$ in [4] and $\xi = \xi_S$ in [1] may lead to large errors in the determination of the heat transfer rate.

Knowing the values of ξ, one can correctly determine the thickness of the viscous sublayer and the transient velocity in a fluid containing solid particles. For this, an equation like (14) is set up and solved with the quantities ξ and χ found for a given stream with a given concentration.

Of special interest is a comparison of δ_0/δ and ξ/ξ_0. According to Fig. 1, the inequality $\xi/\xi_0 > \delta_0/\delta > 1$ is satisfied and this is entirely in agreement with our concepts about the mechanism by which the heat transfer rate increases, namely by the decrease of the boundary-layer thickness as a result of solid particles entering the stream. Here the value of δ_0/δ is somewhat closer to the value in [1]:

$$\frac{\delta_0}{\delta} = \frac{\xi}{\xi_0}, \tag{15}$$

than to the value 1.0 ($\delta = \delta_0$) suggested in [2-5].

In view of this, for a quantitative estimate of the boundary layer, one may use relation (15) with a reasonable error (up to 30%). We must emphasize that it is extremely important here to use expressions (9) and (11) for a correct determination of ξ, because ξ_S/ξ_0 is several times larger than δ_0/δ.

It has been shown, then, that the additivity rule commonly applied to pressure losses in a gaseous suspension stream, according to the original suggestion by Gasterstadt, does not fit the true nature of the process: the presence of solid particles in the stream affects the stresses at the wall, both directly (in terms of ξ_c) and indirectly (through the carrier medium, $\xi \neq \xi_0$).

A proper inclusion of this effect makes it possible to evaluate the thickness of the viscous sublayer and each component of the total pressure loss in a gaseous suspension stream.

NOTATION

v is the absolute velocity;
v^* is the transient velocity;
ν is the kinematic viscosity;
r is the radial coordinate;
d_p is the particle diameter;
D is the channel diameter;
β, μ are the true volume concentration and mass flow concentration respectively;
ξ is the hydraulic drag coefficient;
δ is the thickness of viscous sublayer;
ρ is the density;
m_p is the particle mass;
ν^* is the turbulent analog of viscosity;
Nu is the Nusselt number.

Subscripts

s refers to gas stream;
p refers to solid particle;
0 refers to pure stream, without solid particles;
1 refers to the edge of the viscous sublayer;
c refers to collision;
pw refers to particle-wall collision.

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